Int. J. Heat Mass Transfer. Vol. 36, No. 6, pp. 1485–1493, 1993
Printed in Great Britain

0017-9310/93 $6.00+0.00

Nonsimilarity solutions for mixed convection from vertical surfaces in porous media: variable surface temperature or heat flux

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(*Received* 29 *January* 1992 *and in final form* 20 *July* 1992)

Abstract—Nonsimilarity solutions for mixed convection from a vertical flat plate embedded in a porous medium are reported for two surface heating conditions: variable wall temperature (VWT) and variable surface heat flux (VHF) of the power-law form. The entire mixed convection regime is divided into two regions. One region covers the forced convection dominated regime and the other one covers the free convection dominated regime. The governing equations are first transformed into a dimensionless form by the nonsimilar transformation and then solved by a finite-difference scheme. Four nonsimilarity parameters are introduced. The parameters Ra_x/Pe_x and $Ra_x^*/Pe_x^{3/2}$ characterize the effect of buoyancy forces on the forced convection for the VWT and VHF cases, respectively; while the parameters Pe_x/Ra_x and $Pe_x/Ra_x^{*2/3}$ characterize the effect of forced flow on the free convection for VWT and VHF cases, respectively. Numerical results for both heating conditions are presented. Correlation equations for the local and average Nusselt numbers are also presented.

INTRODUCTION

CONVECTIVE heat transfer along vertical impermeable surfaces in a porous medium has many engineering applications in geothermal reservoirs and petroleum industries. The problem of free convection heat transfer from a vertical flat plate in fluid saturated porous media was studied by Cheng and Minkowycz [1] who obtained similarity solutions for a class of problems where the wall temperature is a power function of the height of the plate. However, the flow and thermal fields in mixed convection from surfaces in a porous medium are nonsimilar, and the local similarity and nonsimilarity solution methods have been employed to obtain solutions for nonsimilar natural and mixed convection problems in porous media [2, 3]. Because some of the higher order terms in the governing equations are neglected in the local similarity and nonsimilarity models, the solutions from these models are approximate in nature. A more accurate solution for nonsimilar boundary layer systems can be obtained by using a finite-difference solution method [4, 5], which was used recently by Aldoss *et al.* [6, 7] to solve the problem of mixed convection over an impermeable horizontal plate in porous media, under the conditions of variable surface heat flux and variable wall temperature in the power-law form.

In the present paper, mixed convection from a vertical flat plate embedded in saturated porous media is analyzed for power-law variation of the wall temperature or power-law variation of the surface heat flux on the plate. In each case, two different transformations are applied to cover the entire mixed convection regime. In the first transformation, the nonsimilarity parameter $\xi_f = Ra_x/Pe_x$ for variable wall temperature (VWT) or $\zeta_f = Ra_x^*/Pe_x^{3/2}$ for variable heat flux (VHF) is found to represent the buoyancy effect in the forced flow dominated mixed convection regime. In the second transformation, the nonsimilarity parameter $\xi_n = Pe_x/Ra_x$ for variable wall temperature (VWT) or $\zeta_n = Pe_x/Ra_x^{*2/3}$ for variable heat flux (VHF) is found to represent the forced flow effect in the buoyancy dominated mixed convection regime. The governing systems of equations are first transformed into a dimensionless form and the resulting equations are then solved by a finite-difference method. Numerical results are obtained for some representative exponent values of the power-law variation in either the wall temperature or the surface heat flux.

ANALYSIS

Consider mixed convection from an impermeable vertical plate embedded in a saturated porous medium. Two surface heating conditions will be considered in the analysis: (1) a power-law variation of the wall temperature, $T_w(x) = T_\infty + ax^n$, and (2) a power-law variation of the surface heat flux, $q_w = bx^m$, where a and b are constants and m and n are the exponents. The x coordinate is measured from the leading edge of the plate and the y coordinate is measured normal to the plate. The gravitational acceleration g is acting downward in the direction opposite to the x coordinate. The Darcy model which is valid under the conditions of low velocities and small pores of porous matrix [8] is used in the analysis. Also, the properties of the fluid are assumed to be constant and

NOMENCLATURE

f, f_1	dimensionless stream functions for the case of VWT
F, F_1	dimensionless stream functions for the case of VHF
$h(x)$	local heat transfer coefficient
$\bar{h}$	average heat transfer coefficient, $(1/L)\int_0^L h(x)\,dx$
k	thermal conductivity
K	permeability coefficient of the porous medium
L	length of the plate
Nu_x	local Nusselt number, hx/k
$\overline{Nu}$	average Nusselt number, $\bar{h}L/k$
Pe_x	local Peclet number, $u_\infty x/\alpha$
q_w	local surface heat flux
Ra_x	local Rayleigh number for the VWT case, $g\beta[T_w(x)-T_\infty]Kx/(\nu\alpha)$
Ra_x^*	modified local Rayleigh number for the VHF case, $g\beta q_w(x)Kx^2/(k\nu\alpha)$
T	temperature
T_∞	free stream temperature
T_w	wall temperature
u, v	velocity components in x- and y-direction
u_∞	free stream velocity
Y, Y_1	pseudo-similarity variables for the VHF case
x, y	axial and normal coordinates.

Greek symbols

α	effective thermal diffusivity of saturated porous medium
β	volumetric coefficient of thermal expansion
δ	boundary layer thickness
η, η_1	pseudo-similarity variables for the VWT case
θ, θ_1	dimensionless temperatures for the VWT case
Θ, Θ_1	dimensionless temperatures for the VHF case
μ	dynamic viscosity
ν	kinematic viscosity
ξ_f, ζ_f	nonsimilarity parameters for the forced convection dominated regime
ξ_n, ζ_n	nonsimilarity parameters for the free convection dominated regime
ρ	fluid density
τ_w	local wall shear stress
ψ	stream function.

Subscripts

f	forced convection dominated condition
n	free convection dominated condition.

the porous medium is treated as isotropic. Under the Boussinesq and the boundary layer approximations, the governing equations can be written as [1]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial^2\psi}{\partial y^2}=\frac{K}{\mu}\rho g\beta\frac{\partial T}{\partial y} \tag{2}$$

$$\alpha\frac{\partial^2 T}{\partial y^2}=u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}. \tag{3}$$

In the equations above, the stream function ψ satisfies the continuity equation (1) with $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$, where u and v are Darcy's velocities in the x and y directions; T is the temperature; ρ, μ and β are the density, dynamic viscosity and thermal expansion coefficient of the fluid; and K and α are, respectively, the permeability and equivalent thermal diffusivity of the porous medium.

The boundary conditions for the present problem are

$$v=0,\ T=T_w(x)=T_\infty+ax^n$$
$$\text{or}\quad q_w=-k(\partial T/\partial y)_{y=0}=bx^m\quad\text{at}\quad y=0$$
$$u\to u_\infty,\ T\to T_\infty\quad\text{as}\quad y\to\infty. \tag{4}$$

Note that $n=0$ corresponds to the case of uniform wall temperature and $m=0$ corresponds to the case of uniform surface heat flux.

Next, the system of equations (2)–(4) will be transformed into dimensionless forms, separately for the case of power-law wall temperature variation and power-law surface heat flux variation.

Power-law variation of wall temperature $T_w(x)=T_\infty+ax^n$

A. Forced convection dominated regime. Equations (2)–(4) can be transformed from the (x, y) coordinates to the dimensionless coordinates $[\xi_f(x), \eta(x, y)]$ by introducing

$$\eta=\frac{y}{x}Pe_x^{1/2},\quad \xi_f=\xi_f(x) \tag{5}$$

$$\psi=\alpha Pe_x^{1/2}f(\xi_f,\eta),\quad \theta(\xi_f,\eta)=\frac{T-T_\infty}{T_w(x)-T_\infty}. \tag{6}$$

Substituting equations (5) and (6) into equations (2)–(4), one can obtain the following system of equations:

$$f''=\xi_f\theta' \tag{7}$$

$$\theta''+\frac{1}{2}f\theta'-nf'\theta=n\xi_f\left(f'\frac{\partial\theta}{\partial\xi_f}-\theta'\frac{\partial f}{\partial\xi_f}\right) \tag{8}$$

with the boundary conditions

$$f(\xi_f,0)+2n\xi_f\frac{\partial f}{\partial \xi_f}(\xi_f,0)=0 \quad \text{or} \quad f(\xi_f,0)=0,$$

$$\theta(\xi_f,0)=1,\quad f'(\xi_f,\infty)=1,\quad \theta(\xi_f,\infty)=0 \quad (9)$$

where

$$\xi_f=\frac{Ra_x}{Pe_x} \quad (10)$$

with $Pe_x=u_\infty x/\alpha$ and $Ra_x=g\beta[T_w(x)-T_\infty]Kx/\nu\alpha$, and the primes denote partial differentiations with respect to η.

In the above system of equations, the parameter ξ_f represents the buoyancy force effect on forced convection. The case of $\xi_f=0$ corresponds to pure forced convection and the limiting case of $\xi_f=\infty$ corresponds to pure free convection.

Some of the physical quantities of interest include the velocity components u and v in the x and y directions, the wall shear stress τ_w, defined as $\tau_w=\mu(\partial u/\partial y)_{y=0}$, and the local Nusselt number $Nu_x=hx/k$, where $h=q_w/[T_w(x)-T_\infty]$. They are given by

$$u=u_\infty f'(\xi_f,\eta) \quad (11)$$

$$v=-\left(\frac{\alpha}{x}\right)Pe_x^{1/2}\left(\frac{1}{2}f-\frac{1}{2}\eta f'+n\xi_f\frac{\partial f}{\partial \xi_f}\right) \quad (12)$$

$$\tau_w=\left(\frac{\mu u_\infty}{x}\right)Pe_x^{1/2}f''(\xi_f,0) \quad (13)$$

$$Nu_x=-Pe_x^{1/2}\theta'(\xi_f,0). \quad (14)$$

The average Nusselt number $\overline{Nu}$ can be evaluated by finding the average heat transfer coefficient $\bar{h}$ from the local Nusselt number expression, equation (14). The final expression is

$$\overline{Nu}=\frac{1}{n}Pe_L^{1/2}\xi_{f_L}^{-1/2n}\int_0^{\xi_{f_L}}[-\theta'(\xi_f,0)]\xi_f^{(1-2n)/2n}\,d\xi_f \quad (15)$$

where Pe_L and ξ_{f_L} are values of Pe_x and ξ_f at $x=L$.

B. Free convection dominated regime. For this case, the following dimensionless variables are introduced in the transformation:

$$\eta_1=\frac{y}{x}Ra_x^{1/2},\quad \xi_n=\xi_n(x) \quad (16)$$

$$\psi=\alpha Ra_x^{1/2}f_1(\xi_n,\eta_1),\quad \theta_1(\xi_n,\eta_1)=\frac{T-T_\infty}{T_w(x)-T_\infty}. \quad (17)$$

Substituting equations (16) and (17) into the governing equations (2)–(4) leads to

$$f_1''=\theta_1' \quad (18)$$

$$\theta_1''+\frac{n+1}{2}f_1\theta_1'-nf_1'\theta_1=n\xi_n\left(\theta_1'\frac{\partial f_1}{\partial \xi_n}-f_1'\frac{\partial \theta_1}{\partial \xi_n}\right) \quad (19)$$

$$(n+1)f_1(\xi_n,0)-2n\xi_n\frac{\partial f_1}{\partial \xi_n}(\xi_n,0)=0 \quad \text{or} \quad f_1(\xi_n,0)=0,$$

$$\theta_1(\xi_n,0)=1,\quad f_1'(\xi_n,\infty)=\xi_n,\quad \theta_1(\xi_n,\infty)=0 \quad (20)$$

where

$$\xi_n=\frac{Pe_x}{Ra_x} \quad (21)$$

and the primes in equations (18)–(20) denote partial differentiations with respect to η_1.

Note that the ξ_n parameter here represents the forced flow effect on free convection. The case of $\xi_n=0$ corresponds to pure free convection and the limiting case of $\xi_n=\infty$ corresponds to pure forced convection.

The velocity components u and v, the wall shear stress, and the local Nusselt number for this case have the following expressions

$$u=\frac{\alpha}{x}Ra_xf_1'(\xi_n,\eta_1) \quad (22)$$

$$v=-\left(\frac{\alpha}{x}\right)Ra_x^{1/2}\left(\frac{n+1}{2}f_1+\frac{n-1}{2}\eta_1f_1'-n\xi_n\frac{\partial f_1}{\partial \xi_n}\right) \quad (23)$$

$$\tau_w=\left(\frac{\mu\alpha}{x^2}\right)Ra_x^{3/2}f_1''(\xi_n,0) \quad (24)$$

$$Nu_x=-Ra_x^{1/2}\theta_1'(\xi_n,0). \quad (25)$$

The average Nusselt number $\overline{Nu}$ is

$$\overline{Nu}=-\frac{1}{n}Ra_L^{1/2}\xi_{n_L}^{(n+1)/2n}\times\int_0^{\xi_{n_L}}[-\theta_1'(\xi_n,0)]\xi_n^{-(3n+1)/2n}\,d\xi_n \quad (26)$$

where Ra_L and ξ_{n_L} are values of Ra_x and ξ_n at $x=L$.

It is noted that the solutions of the two systems of equations, equations (7)–(9) and (18)–(20), will cover the entire mixed convection regime. The relationship between ξ_f and ξ_n is $\xi_n=\xi_f^{-1}$.

Power-law variation of surface heat flux, $q_w(x)=bx^m$

A. Forced convection dominated regime. For this case, let the dimensionless variables be

$$Y=\frac{y}{x}Pe_x^{1/2},\quad \zeta_f=\zeta_f(x) \quad (27)$$

$$\psi=\alpha Pe_x^{1/2}F(\zeta_f,Y),\quad \Theta(\zeta_f,Y)=\frac{(T-T_\infty)Pe_x^{1/2}}{q_w(x)x/k}. \quad (28)$$

Substitution of equations (27) and (28) into equations (2)–(4) then yields

$$F'' = \zeta_f \Theta' \tag{29}$$

$$\Theta'' + \frac{1}{2}F\Theta' - \left(m+\frac{1}{2}\right)F'\Theta = \left(m+\frac{1}{2}\right)\zeta_f\left(F'\frac{\partial\Theta}{\partial\zeta_f} - \Theta'\frac{\partial F}{\partial\zeta_f}\right) \tag{30}$$

$$F(\zeta_f,0) + (2m+1)\zeta_f\frac{\partial F}{\partial\zeta_f}(\zeta_f,0) = 0 \quad \text{or} \quad F(\zeta_f,0) = 0,$$

$$\Theta'(\zeta_f,0) = -1, \quad F'(\zeta_f,\infty) = 1, \quad \Theta(\zeta_f,\infty) = 0 \tag{31}$$

where

$$\zeta_f = \frac{Ra_x^*}{Pe_x^{3/2}} \tag{32}$$

with $Ra_x^* = g\beta q_w(x)Kx^2/k\nu\alpha$ and the primes now denote partial differentiations with respect to Y.

The nonsimilar parameter ζ_f represents the buoyancy force effect on forced convection. The case of $\zeta_f = 0$ corresponds to pure forced convection and the limiting case of $\zeta_f = \infty$ corresponds to pure free convection.

The velocity components u and v, the wall shear stress, and the local Nusselt number for this case have the expressions

$$u = u_\infty F'(\zeta_f, Y) \tag{33}$$

$$v = -\left(\frac{\alpha}{x}\right)Pe_x^{1/2}\left[\frac{1}{2}F - \frac{1}{2}YF' + \left(m+\frac{1}{2}\right)\zeta_f\frac{\partial F}{\partial\zeta_f}\right] \tag{34}$$

$$\tau_w = \left(\frac{\mu u_\infty}{x}\right)Pe_x^{1/2}F''(\zeta_f,0) \tag{35}$$

$$Nu_x = \frac{Pe_x^{1/2}}{\Theta(\zeta_f,0)}. \tag{36}$$

The average Nusselt number $\overline{Nu}$ can be expressed as

$$\overline{Nu} = \frac{2}{2m+1}Pe_L^{1/2}\zeta_{f_L}^{-1/(2m+1)}\int_0^{\zeta_{f_L}}\frac{\zeta_f^{-2m/(2m+1)}}{\Theta(\zeta_f,0)}\,d\zeta_f \tag{37}$$

where Pe_L and ζ_{f_L} are values of Pe_x and ζ_f at $x = L$.

B. Free convection dominated regime. In this regime, one introduces the dimensionless variables

$$Y_1 = \frac{y}{x}Ra_x^{*1/3}, \quad \zeta_n = \zeta_n(x) \tag{38}$$

$$\psi = \alpha Ra_x^{*1/3}F_1(\zeta_n, Y_1),$$

$$\Theta_1(\zeta_n, Y_1) = \frac{(T-T_\infty)Ra_x^{*1/3}}{q_w(x)x/k}. \tag{39}$$

The transformation of equations (2)–(4) then results in

$$F_1'' = \Theta_1' \tag{40}$$

$$\Theta_1'' + \frac{1}{3}(m+2)F_1\Theta_1' - \frac{1}{3}(2m+1)F_1'\Theta_1 = \frac{1}{3}(2m+1)\zeta_n\left(\Theta_1'\frac{\partial F_1}{\partial\zeta_n} - F_1'\frac{\partial\Theta_1}{\partial\zeta_n}\right) \tag{41}$$

$$(m+2)F_1(\zeta_n,0) - (2m+1)\zeta_n\frac{\partial F_1}{\partial\zeta_n}(\zeta_n,0) = 0$$

$$\text{or} \quad F_1(\zeta_n,0),$$

$$\Theta_1'(\zeta_n,0) = -1, \quad F_1'(\zeta_n,\infty) = \zeta_n, \quad \Theta_1(\zeta_n,\infty) = 0 \tag{42}$$

where

$$\zeta_n = \frac{Pe_x}{Ra_x^{*2/3}} \tag{43}$$

and the primes denote partial differentiations with respect to Y_1.

The ζ_n parameter here represents the forced flow effect on free convection. The case of $\zeta_n = 0$ corresponds to pure free convection and the limiting case of $\zeta_n = \infty$ corresponds to pure forced convection.

The velocity components u and v, the wall shear stress, and the local Nusselt number are now given, respectively, by

$$u = \frac{\alpha}{x}Ra_x^{*2/3}F_1'(\zeta_f, Y_1) \tag{44}$$

$$v = -\left(\frac{\alpha}{x}\right)Ra_x^{*1/3}\left[\frac{1}{3}(m+2)F_1 + \frac{1}{3}(m-1)Y_1F_1' - \frac{1}{3}(2m+1)\zeta_n\frac{\partial F_1}{\partial\zeta_n}\right] \tag{45}$$

$$\tau_w = \left(\frac{\mu\alpha}{x^2}\right)Ra_x^*F_1''(\zeta_n,0) \tag{46}$$

$$Nu_x = \frac{Ra_x^{*1/3}}{\Theta_1(\zeta_n,0)}. \tag{47}$$

The average Nusselt number $\overline{Nu}$ is

$$\overline{Nu} = -\frac{3}{2m+1}Ra_L^{*1/3}\zeta_{n_L}^{(m+2)/(2m+1)} \times \int_0^{\zeta_{n_L}}\frac{\zeta_n^{-(3m+3)/(2m+1)}}{\Theta_1(\zeta_n,0)}\,d\zeta_n \tag{48}$$

where Ra_L^* and ζ_{n_L} are values of Ra_x^* and ζ_n at $x = L$.

A combination of the above two treatments will then cover the entire regime of mixed convection. The relationship between ζ_f and ζ_n is $\zeta_n = \zeta_f^{-2/3}$.

The systems of equations for the forced convection dominated regime under the power-law variation of wall temperature, equations (7)–(9), and under the power-law variation of surface heat flux, equations (29)–(31), are valid for $0 \leqslant \xi_f < \infty$ and $0 \leqslant \zeta_f < \infty$. They were solved by the finite-difference method as described by Cebeci and Bradshaw [4]. However, there was difficulty in finding the convergent solutions of equations for the free convection dominated regime for both heating conditions, i.e. equations (18)–(20)

and (40)–(42). Thus, for the case of free convection dominated regime, solutions were obtained only for pure free convection from the system of equations (18)–(20) or (40)–(42) with $\xi_n = 0$ or $\zeta_n = 0$ by the Runge–Kutta numerical integration scheme. A combination of the two solutions, one for the forced convection dominated regime and the other for pure free convection, then covers the entire convection regime for both power-law variations of the wall temperature and the surface heat flux.

RESULTS AND DISCUSSION

Representative numerical results for both cases of variable wall temperature and variable wall heat flux will be illustrated and discussed in this section. The range of n or m values for which the present problem is physically realistic can be found following the argument used by Cheng and Minkowycz [1]. Since the wall temperature begins to deviate from T_∞ at $x = 0$ and convective flow must start at this point, both u and δ, the streamwise velocity component and the boundary layer thickness, must increase or at least remain constant with respect to x. From equations (22) and (44) one finds that u varies with x^n or $x^{(2m+1)/3}$. Also, from equations (16) and (38) the boundary layer thickness δ, which is of the order of y, varies with $x^{(1-n)/2}$ or $x^{(1-m)/3}$. Thus, the above conditions can be satisfied if $0 \leqslant n \leqslant 1$ for the variable wall temperature case or $-0.5 \leqslant m \leqslant 1$ for the variable surface heat flux case. The numerical solutions were carried out for values of n and m within the above range.

Power-law variation of wall temperature

Results for the temperature and velocity profiles, $\theta(\xi_f, \eta)$ and $f'(\xi_f, \eta)$, are shown in Figs. 1 and 2 for different values of ξ_f and n. Figure 1 shows that for a given value of ξ_f the thermal boundary layer thickness decreases and the temperature gradient at the wall increases when n increases, resulting in a higher heat transfer rate at a higher value of n. In addition, the

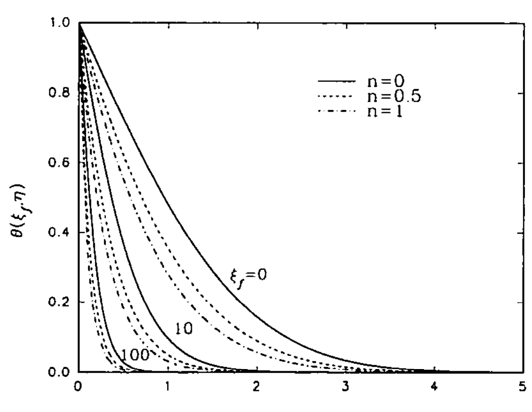

FIG. 1. Dimensionless temperature profiles at selected values of ξ_f and n (VWT case).

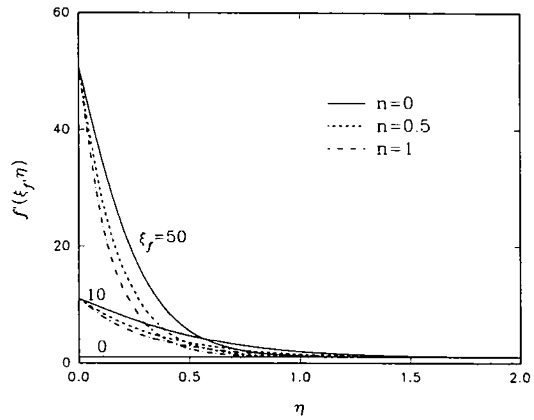

FIG. 2. Dimensionless velocity profiles at selected values of ξ_f and n (VWT case).

wall temperature gradient and hence the heat transfer rate is seen to increase with an increase in ξ_f for a particular value of n. Figure 2 shows that for a given value of n, an increase in the buoyancy parameter ξ_f increases the slip velocity at the wall. Also, as n increases the momentum boundary layer thickness decreases for a given value of ξ_f. It should be mentioned here that from equation (7) and boundary condition (9), one can obtain

$$f' = \xi_f \theta + 1 \tag{49}$$

and

$$f'(\xi_f, 0) = \xi_f + 1. \tag{50}$$

Figure 2 shows that the predicted results agree well with equation (50).

To find the local Nusselt number and the local wall shear stress, one needs to know the values of $-\theta'(\xi_f, 0)$ and $f''(\xi_f, 0)$. These quantities at selected values of ξ_f are listed in Table 1 for different values of n. The local Nusselt numbers Nu_x in terms of $Nu_x Pe_x^{-1/2}$ for values of the exponent n of 0, 0.5, and 1 are shown in Fig. 3 for the entire mixed convection regime. From Fig. 3, it is seen that a higher Nusselt number occurs at higher values of n and ξ_f. This implies that the stronger the buoyancy force, the larger the surface heat transfer rate will be. The domains of pure forced convection, mixed convection and pure free convection can be established from the present results based on a 5% departure in the local Nusselt number from the pure forced convection limit and from the pure free convection limit. They are listed in Table 2.

For practical purposes, correlation equations were developed for the local Nusselt numbers. By using the cubic spline interpolation technique the local Nusselt number for pure forced convection in the range of $0 \leqslant n \leqslant 1$ can be correlated by

$$Nu_f = g_1(n) Pe_x^{1/2} \tag{51}$$

where

$$g_1(n) = 0.5650 + 0.7631n - 0.2813n^2 + 0.0821n^3. \tag{52}$$

Table 1. Values of $-\theta'(\xi_f, 0)$ and $f''(\xi_f, 0)$ at selected values of ξ_f for different n values (VWT case)

	$-\theta'(\xi_f, 0)$			$f''(\xi_f, 0)$		
$\xi_f = Ra_x/Pe_x$	$n = 0$	$n = 0.5$	$n = 1.0$	$n = 0$	$n = 0.5$	$n = 1.0$
0	0.5642	0.8862	1.1284	0	0	0
0.5	0.6474	1.0428	1.3339	−0.3237	−0.5215	−0.6670
1.0	0.7206	1.1780	1.5109	−0.7206	−1.1780	−1.5109
10	1.5163	2.5960	3.3591	−15.163	−25.860	−33.591
20	2.0665	3.5602	4.6128	−41.329	−71.204	−92.256
30	2.4981	4.3140	5.5924	−74.944	−129.42	−167.77
40	2.8665	4.9545	6.4244	−114.62	−198.18	−256.98
50	3.1909	5.5212	7.1605	−159.54	−276.06	−358.02
100	4.4763	7.7570	10.063	−447.62	−775.70	−1006.3
500	9.9555	17.264	22.389	−4977.7	−8631.9	−11195
1000	14.090	24.419	31.643	−14090	−24419	−31643
$\xi_n = Pe_x/Ra_x$	$-\theta_1'(\xi_n, 0)$			$f_1''(\xi_n, 0)$		
0 ($\xi_f = \infty$)	0.4438	0.7704	1.0000	−0.4438	−0.7704	−1.0000

Table 2. Range of ξ_f values for pure forced convection, mixed convection, and pure free convection (VWT case)

Exponent	Range of $\xi_f = Ra_x/Pe_x$ values		
n	Forced convection	Mixed convection	Free convection
0	0–0.16	0.16–16.4	16.4–∞
0.5	0–0.13	0.13–13.3	13.3–∞
1.0	0–0.12	0.12–12.5	12.5–∞

For the case of pure free convection, the corresponding correlation equation for the local Nusselt number is given by

$$Nu_n = g_2(n) Ra_x^{1/2} \tag{53}$$

where

$$g_2(n) = 0.4457+0.8099n-0.3831n^2+0.1286n^3. \tag{54}$$

Equations (51) and (53) fit the computed results for the pure forced and pure free convection within an error of less than 2% respectively.

Following Churchill [9] the correlation equation for the local Nusselt number in mixed convection can be expressed as

$$\left(\frac{Nu_x}{Nu_f}\right)^p = 1+\left(\frac{Nu_n}{Nu_f}\right)^p. \tag{55}$$

For the present problem the correlation equation for

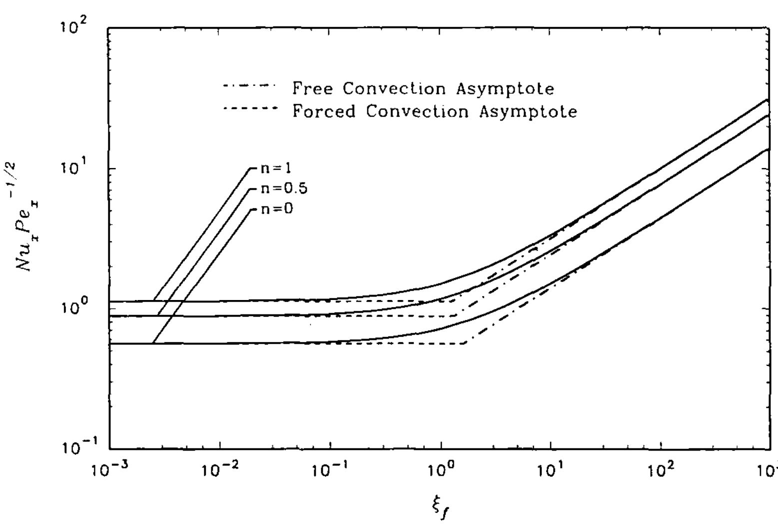

FIG. 3. Local Nusselt number variation for mixed convection with variable wall temperature (VWT).

the local mixed convection Nusselt number can be represented by

$$\frac{Nu_x Pe_x^{-1/2}}{g_1(n)} = \left\{1+\left[\frac{g_2(n)(Ra_x/Pe_x)^{1/2}}{g_1(n)}\right]^p\right\}^{1/p}. \quad (56)$$

The average Nusselt numbers $\overline{Nu}$ for pure forced convection and pure free convection are found to be

$$\overline{Nu}_f = 2g_1(n)Pe_L^{1/2} \quad (57)$$

$$\overline{Nu}_n = \frac{2}{n+1}\,g_2(n)Ra_L^{1/2} \quad (58)$$

where $\overline{Nu}_f$ and $\overline{Nu}_n$ are the expressions of $\overline{Nu}$ for pure forced convection and pure free convection, respectively.

The corresponding correlation equation for the average mixed convection Nusselt number $\overline{Nu}$ can be expressed by

$$\frac{\overline{Nu}Pe_L^{-1/2}}{2g_1(n)} = \left\{1+\left[\frac{2g_2(n)(Ra_L/Pe_L)^{1/2}}{2(n+1)g_1(n)}\right]^p\right\}^{1/p}. \quad (59)$$

With an exponent value of $p = 2$ equations (56) and (59) are found to correlate very well with the predicted results, respectively from equations (14) and (15), giving a maximum deviation of less than 0.4% between the correlated and the predicted mixed convection Nusselt numbers for the range of $0 \leqslant n \leqslant 1$ over the entire regime of mixed convection. When $p = 3$ is used, the maximum deviation between the correlated and predicted values is about 10%.

Power-law variation of surface heat flux

The results for $\Theta(\zeta_f,\eta)/\Theta(\zeta_f,0)$ and $F'(\zeta_f,\eta)$, the temperature and velocity profiles, for $-0.5 \leqslant m \leqslant 1$ are illustrated in Figs. 4 and 5. The behaviors of the temperature and velocity profiles for the variable surface heat flux case are similar to those for the variable wall temperature case presented earlier.

The values of $1/\Theta(\zeta_f,0)$ and $F''(\zeta_f,0)$ for the various m values are listed in Table 3 for selected ζ_f values. One can see from Table 3 that the value of $F''(\zeta_f,0)$ is

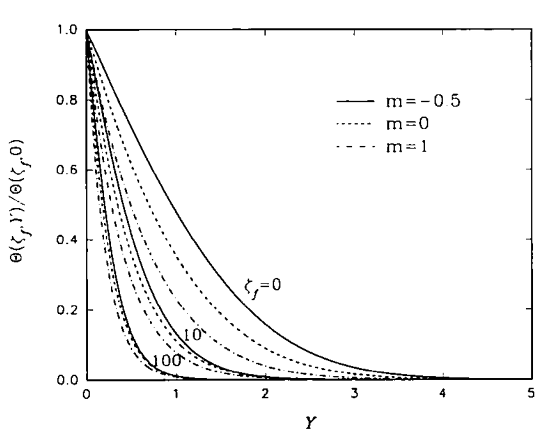

FIG. 4. Dimensionless temperature profiles at selected values of ζ_f and m (VHF case).

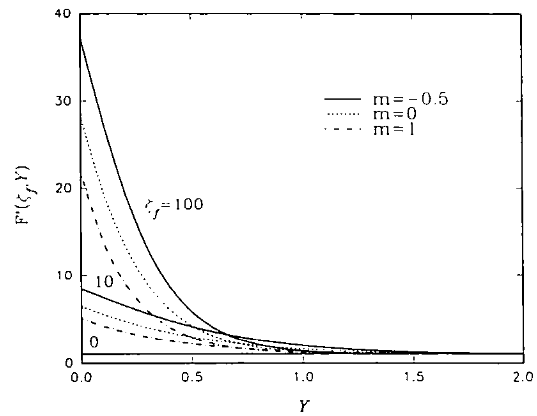

FIG. 5. Dimensionless velocity profiles at selected values of ζ_f and m (VHF case).

exactly equal to the value of $-\zeta_f$ and is not a function of m. This is so, because from equation (29) and boundary condition (31) one can obtain

$$F''(\zeta_f,0) = -\zeta_f. \quad (60)$$

Local Nusselt numbers in terms of $Nu_x Pe_x^{-1/2}$ for different values of the exponent m are shown in Fig. 6. The trends and behaviors of these curves are similar to those described for the case of variable wall temperature, because the buoyancy and forced flow effects between the two cases are similar. The domains for pure forced convection, mixed convection and pure free convection are shown in Table 4. Calculations of these values are also based on a 5% departure in the local Nusselt number from pure forced convection or pure free convection limit.

The local Nusselt number correlation equations for pure forced convection and pure free convection in the range $-0.5 \leqslant m \leqslant 1$ are given by

$$Nu_f = g_3(m)Pe_x^{1/2} \quad (61)$$

$$Nu_n = g_4(m)Ra_x^{*\,1/3} \quad (62)$$

where

$$g_3(m) = 0.8864+0.5488m-0.1559m^2+0.0516m^3 \quad (63)$$

$$g_4(m) = 0.7718+0.3043m-0.1189m^2+0.0444m^3. \quad (64)$$

Equations (61) and (62) fit the computed results for pure forced and pure free convection within an error of less than 2%.

The correlation equation for the local Nusselt number in mixed convection can be represented by

$$\frac{Nu_x Pe_x^{-1/2}}{g_3(m)} = \left\{1+\left[\frac{g_4(m)(Ra_x^*/Pe_x^{3/2})^{1/3}}{g_3(m)}\right]^p\right\}^{1/p} \quad (65)$$

and the average Nusselt numbers for pure forced, pure free and mixed convection can be expressed, respectively, by

Table 3. Values of $1/\Theta(\zeta_f, 0)$ and $F''(\zeta_f, 0)$ at selected values of ζ_f for different m values (VHF case)

	$1/\Theta(\zeta_f, 0)$				$F''(\zeta_f, 0)$
$\zeta_f = Ra_x^*/Pe_x^{3/2}$	$m = -0.5$	$m = 0$	$m = 0.5$	$m = 1.0$	all m
0	0.5642	0.8863	1.1284	1.3294	0
0.5	0.6825	1.0099	1.2550	1.4579	−0.5
1.0	0.7625	1.1017	1.3530	1.5601	−1.0
10	1.3401	1.8252	2.1658	2.4386	−10
20	1.6482	2.2245	2.6245	2.9425	−20
30	1.8680	2.5114	2.9556	3.3074	−30
40	2.0445	2.7425	3.2227	3.6023	−40
50	2.1942	2.9389	3.4502	3.8537	−50
100	2.7411	3.6581	4.2843	4.7768	−100
500	4.6298	6.1234	7.2054	8.0165	−500
1000	5.8361	7.7150	8.9981	10.063	−1000
$\zeta_n = Pe_x/Ra_x^{*2/3}$	$1/\Theta_1(\zeta_n, 0)$				$F_1''(\zeta_n, 0)$
0 ($\zeta_f = \infty$)	0.5818	0.7715	0.8998	1.0000	−1.0

Table 4. Range of ζ_f values for pure forced convection, mixed convection, and pure free convection (VHF case)

Exponent m	Range of $\zeta_f = Ra_x^*/Pe_x^{3/2}$ values: Forced convection	Mixed convection	Free convection
−0.5	0–0.09	0.09–16.8	16.8–∞
0	0–0.15	0.15–27.9	27.9–∞
0.5	0–0.20	0.20–36.8	36.8–∞
1.0	0–0.24	0.24–44.2	44.2–∞

$$\overline{Nu}_f = 2g_3(m)Pe_L^{1/2} \tag{66}$$

$$\overline{Nu}_n = \frac{3}{m+2}\, g_4(m)Ra_L^{*\,1/3} \tag{67}$$

$$\frac{\overline{Nu}Pe_L^{-1/2}}{2g_3(m)} = \left\{1+\left[\frac{3g_4(m)(Ra_L^*/Pe_L^{3/2})^{1/3}}{2(m+2)g_3(m)}\right]^p\right\}^{1/p}. \tag{68}$$

When $p = 3$ is used, equations (65) and (68) correlate very well with the predicted numerical results, respectively from equations (36) and (37), with a maximum deviation of less than 5% between the predicted and correlated values for the range $-0.5 \leqslant m \leqslant 1$ over the entire regime of mixed convection. However, the maximum deviation increases to about 7% when $p = 2$ is used.

To the best knowledge of the authors, there are no

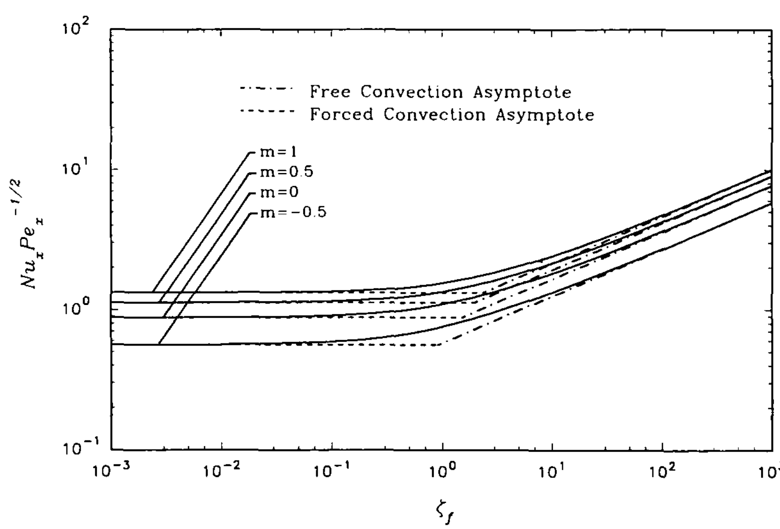

FIG. 6. Local Nusselt number variation for mixed convection with variable surface heat flux (VHF).

experimental data reported for mixed convection in boundary layer flow along a vertical plate in porous media. However, the present results for pure free convection (i.e. $\xi_n = 0$) under uniform wall temperature can be compared with the experimental work of Cheng *et al.* [10]. The predicted local Nusselt number $Nu_x Ra_x^{-1/2}$ for $\xi_n = 0$ and $n = 0$ listed in Table 1 agrees well with the experimental data when Ra_x is less than 500. It may then be concluded that the non-Darcian effect will become important when $Ra_x \geqslant 500$. In the absence of experimental data, the present results based on Darcy's law are expected to be valid for Pe_x and Ra_x that are smaller than 1000.

CONCLUDING REMARKS

In this paper, mixed convection from a vertical flat plate in saturated porous media has been studied analytically for two surface heating conditions, power-law variation in the wall temperature and power-law variation in the surface heat flux. Numerical results are presented for both heating conditions. They include dimensionless temperature and velocity profiles, and Nusselt numbers. A 5% rule is used to establish the regime where mixed convection becomes important for the various surface heating conditions. General correlation equations for the local and average Nusselt numbers are also developed for the entire regime of mixed convection. The correlation equations agree well with calculated numerical results within a maximum deviation of less than 5%.

Acknowledgement—The present study was supported in part by a grant from the University of Missouri (Weldon Spring/Chen/91–92).

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